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# INFLUENCE OF NORMAL PRESSURE ON THE NATURAL-OSCILLATION FREQUENCIES OF NEARLY CYLINDRICAL SHELLS

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# INFLUENCE OF NORMAL PRESSURE ON THE NATURAL-OSCILLATION FREQUENCIES OF NEARLY CYLINDRICAL SHELLS

S. Shukurov

*ABSTRACT: The dynamic behavior of thin isotropic, nearly cylindrical shells under static normal loads is discussed in this article. The effect of deviations from a perfect cylindrical form on the natural oscillations of such shells is determined. A solution is obtained by a small-parameter method previously used for static problems and stability problems of such shells. The maximum deviation of the shell from a perfect form is assumed to be the small parameter in this procedure.*

## 1. Basic Equations

Let $z$ and $\phi$ be coordinates of a point on the middle surface of the shell, where $z$ is the distance from this point to a certain initial one and $\phi$ is the angle between an axial plane passing through this point and the fixed axial plane. The meridian of nearly cylindrical shells is described by the following equation: /72*

$$r = R[1 - \lambda f(z)]. \tag{1.1}$$

The coefficients of the first quadratic form and the principal curvatures of the surface in (1.1) can now be written in the following way:

$$A = 1 + \frac{1}{2}\lambda^2 R^2 f'^2;\ B = R(1 - \lambda f);$$
$$k_1 = 1/R_1 = \lambda R f'';\ k_2 = 1/R_2 = \frac{1 + \lambda f + \lambda^2\left(f^2 - \frac{1}{2}R^2 f'^2\right)}{R}. \tag{1.2}$$

Here $R$ is the radius of the cylinder; $f(z)$ is the function determining the form of the shell meridian; $\lambda$ is the small parameter whose cubes and higher powers will be disregarded.

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*Numbers in the margin indicate pagination in the foreign text.

See A68-26659 for original Text.

Let a freely oscillating, nearly cylindrical shell first support normal forces which are distributed uniformly over the lateral face with intensity . It is assumed that a zero-moment strained state is realized in the shell by the effect of the pressure (Fig. 1).

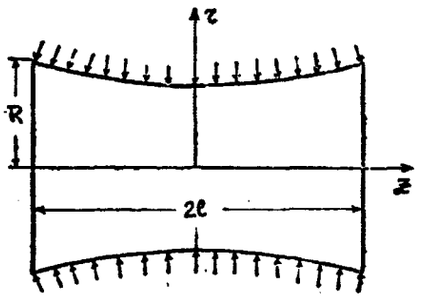


Fig. 1. Nearly Cylindrical Shell.

In solving the problem, we will base our discussions on the system of equations in (2.1) presented in [7], adding to the left-hand part of the third equation the following load factor:

$$Q = T_1^0 \varkappa_1 + T_2^0 \varkappa_2. \qquad (1.3)$$ /73

The differential operators of $L_{ki}(k = 1,2;\ i = 0,1,2)$ of the system in (2.1) will remain the same as in [7], and the load factors corresponding to the forces in (1.3) will be added to $L_{3i}$:

$$Q = \frac{p}{R}\left\{\frac{\partial^2 w}{\partial \tau^2} + \lambda\left(R^2 f \frac{\partial^2 w}{\partial z^2} + f\frac{\partial^2 w}{\partial \tau^2} - R^2 f' \frac{\partial w}{\partial z}\right) + \lambda^2\left[\frac{1}{2} R^2 f^2 \frac{\partial^2 w}{\partial z^2} + \right.\right.$$
$$\left.\left. + (f^2 + ff'' + \frac{1}{2} f'^2)\frac{\partial^2 w}{\partial \tau^2}\right]\right\}.$$

We will look for the components of the dislocations $u$, $v$, $w$ and the pressure $p$ in the following forms:

$$u = \lambda^i u_i;\ v = \lambda^i v_i;\ w = \lambda^i w_i;\ p = \lambda^i p_i, \qquad (1.4)$$

Moreover, we will consider that $u_i$, $v_i$, $w_i$ and $p_i$ $(i = 0,1,2)$ do not depend on $\lambda$.

Substituting (1.4) into the original oscillation equations, we find the following:

$$\lambda^i \sum_{j=0}^{i} L_{kj}(u_{i-j},\ v_{i-j},\ w_{i-j}) = 0,\quad (i = 0,\ 1,\ 2;\ k = 1,\ 2, \qquad (1.5)$$

where the index $i$ means summation, as in [7].

Given the components of the dislocations $u_i$, $v_i$ and $w_i$ which satisfy the boundary conditions for free support by the end faces

$z = \pm l$, and substituting them into (1.5), we will integrate the equations obtained by the Bubnov-Galerkin method. As a result of integration, we arrive at a uniform system of three algebraic equations. Having made this system equal to zero, we will obtain the equation for the frequency of natural oscillations of the shells under investigation.

## 2. Oscillation of Concave Shells

The method presented in the first section aided in solving the problem of determining the effect of a preliminary strained state due to normal pressure on the spectrum for the frequencies of natural oscillations of nearly cylindrical shells.

The deviation of the meridian of the shell under investigation from a rounded cylinder is taken in the following way:

$$f(z) = 1 - z^2/l^2. \tag{2.1}$$

The components of the dislocations $u_i$, $v_i$, $w_i (i = 0,1,2)$ will be taken in the form of (2.7) and (3.1) in [7]. In the case of external pressure, we can consider that the number of half-waves along the generating line is equal to one.

If we disregard the tangential components of the inertial forces as compared to the normal one, we will obtain the following expression for the square of the frequency of natural oscillations of shells which are close to cylindrical ones:

$$\omega^2 = \omega_0^2\left[1 - \frac{q_{10}}{F_{10}} + \lambda\left\{\frac{F_{11}+q_{11}}{F_{10}} - \frac{F_{21}}{F_{20}}\left(1 + \frac{q_{10}}{F_{10}}\right)\right\} + \right.$$
$$\left. + \lambda^2\left\{\frac{F_{12}+q_{12}}{F_{10}} - \frac{F_{22}}{F_{20}}\left(1 + \frac{q_{10}}{F_{10}}\right) - \frac{F_{21}}{F_{20}}\left[\frac{F_{11}+q_{11}}{F_{10}} - \frac{F_{21}}{F_{20}}\left(1 + \frac{q_{10}}{F_{10}}\right)\right]\right\}\right]. \tag{2.2}$$

Here

$$F_{10} = \mu^4 + \varepsilon(\mu^2 + n^2)^4,\ \mu = \frac{m\pi R}{2l},\ \varepsilon = \frac{h^2}{3(1-\sigma^2)R^2};$$

$$F_{11} = \frac{2(5+3\sigma)}{3}\mu^4 + \frac{6-\sigma-8n^2}{2}\mu^2 r^2 + \varepsilon(\mu^2 + n^2)^2\left[\frac{2\sigma}{3}(\mu^2 + n^2)^2 + \right.$$
$$\left. + \frac{8}{3}n^2(\mu^2 + n^2) + \frac{(4+\sigma)(\mu^2+n^2)^2 - 8(1-\sigma)\mu^4}{2}\cdot\frac{r^2}{\mu^2}\right],\ r^2 = R^2/l^2;$$

$$F_{12} = \frac{(327 + 332\sigma - 229\sigma^2 - 336\sigma^3 - 93\sigma^4)\mu^4 + 2(\sigma^3 - 2\sigma^2 + \sigma + 1)\mu^2 n^2 + \sigma^2 n^4}{45(1-\sigma^2)} + \tag{2.3}$$

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$$+ \frac{1}{6(1-\sigma^2)}\cdot\frac{r^2}{\mu^2}[(88 + 66\sigma - 46\sigma^2 - 64\sigma^3 - 15\sigma^4)\mu^4 +$$
$$+ (3\sigma^2 + 14\sigma + 6)n^4 + (157 + 93\sigma - 23\sigma^2 - 11\sigma^3)\mu^2 n^2] +$$

$$+\frac{1}{16(1-\sigma^2)}\cdot\frac{r^4}{\mu^4}[(23\sigma^4+60\sigma^3-38\sigma^2-98\sigma+56)\mu^4+$$

$$+2(27\sigma^2+64\sigma-28)n^4+2(15\sigma^3+46\sigma^2-89\sigma-187)\mu^2n^2]-$$

$$-\frac{2}{3}r^2[5\mu^4+12(2+\sigma)\mu^2n^2]+r^4[5\mu^2-2(3-\sigma)n^2+4n^4]+$$

$$+\varepsilon(\mu^2+n^2)\left\{\frac{2}{45}[\sigma(30+23\sigma)(\mu^2+n^2)^2+24(8+7\sigma)n^2(\mu^2+n^2)+\right.$$

$$+208n^4]+\frac{2}{3}\frac{r^2}{\mu^2}[2(26\sigma^2+27\sigma-16)(\mu^2+n^2)^2+(32+10\sigma-$$

$$-27\sigma^2)\mu^2n^2+(22-10\sigma-13\sigma^2)n^4+\mu^2(\mu^2+n^2)(n^2-3\mu^2)]+$$

$$\left.+\frac{r^4}{\mu^4}\left[\frac{39\sigma^2+64\sigma-8-8\mu^2}{8}(\mu^2+n^2)^2-2(6\sigma^2+11\sigma-6)\mu^2n^2-\right.\right. \qquad (2.3)$$

$$\left.\left.-(5\sigma^2+4\sigma-16)n^4\right]\right\}-\frac{8}{3}\varepsilon n^2r^2[8\sigma n^2(\mu^2+n^2)-$$

$$-3r^2(1-\sigma^2)(3\mu^2+n^2)];$$

$$F_{20}=(\mu^2+n^2)^2;\quad F_{21}=2(1+\sigma)(\mu^2+n^2)^2+\frac{8}{3}n^2(\mu^2+n^2)+$$

$$+\frac{(4+3\sigma)(\mu^2+n^2)^2-4(1+\sigma)n^2(3\mu^2+n^2)}{2}\cdot\frac{r^2}{\mu^2};$$

$$F_{22}=\frac{94\sigma^2+204\sigma+135}{45}(\mu^2+n^2)^2+\frac{2(193-125\sigma-5\sigma^2-\sigma^3)}{45}\mu^2n^2+$$

$$+\frac{16(29+16\sigma)}{45}n^4+\frac{r^2}{\mu^2}\left[\frac{2(8\sigma^2+15\sigma+12+\mu^2)}{3}(\mu^2+n^2)^2-\right.$$

$$\left.-\frac{11\sigma^3+53\sigma^2+73\sigma-5}{6}\mu^2n^2+\frac{2(2\sigma^2+8\sigma+15)}{3}n^4-\frac{8}{3}\mu^4(\mu^2+n^2)\right]+$$

$$+\frac{r^4}{\mu^4}\left[\frac{5\sigma(4+3\sigma)-8(2+\mu^2)}{8}(\mu^2+n^2)^2-(5\sigma^2+11\sigma+8)n^4+\right.$$

$$\left.+\frac{(\sigma^3-81\sigma^2-237\sigma-171)\mu^2n^2}{8}\right], \qquad (2.4)$$

$$q_{10}=S_{10}\bar{q}_{10};\quad q_{11}=S_{11}\bar{q}_{10}+S_{10}\bar{q}_{11};\quad q_{12}=S_{12}\bar{q}_{10}+S_{11}\bar{q}_{11}+S_{10}\bar{q}_{12},$$

$$\bar{q}_{10}=-\frac{p_0R}{2Eh}n^2;\quad S_{10}=(\mu^2+n^2)^2;\quad \bar{q}_{11}=-\frac{R}{2Eh}(p_0\psi_{mn}+p_1n^2),\quad \psi_{mn}=$$

$$=\frac{2(1+\sigma)n^2-2\mu^2}{3}+\frac{\mu^2+(1+\sigma)n^2}{2}\cdot\frac{r^2}{\mu^2};$$

$$\bar{q}_{12}=-\frac{p_0R}{2Eh}\left\{\frac{2(2+\sigma)^2n^2-4(1+2\sigma)\mu^2}{15}-\frac{3}{8}\frac{r^4}{\mu^4}\left[2\mu^2-\right.\right. \qquad (2.5)$$

$$\left.\left.-(2+\sigma)^2n^2+8\mu^2n^2\right]\right\}-\frac{R}{2Eh}(p_1\psi_{mn}+p_2n^2);$$

$$S_{11} = \frac{2(8+2\sigma)}{3}(\mu^2+n^2)^2 + \frac{8}{3}n^2(\mu^2+n^2)^2 +$$

$$+ \frac{r^2}{\mu^2}[(2+\sigma)(\mu^2+n^2)^2 - 2(1+\sigma)n^2(3\mu^2+n^2)],$$

$$S_{12} = \frac{16\sigma^2+48\sigma+45}{15}(\mu^2+n^2)^2 + \frac{16(29+11\sigma)}{45}n^4 - \frac{2(\sigma^3+5\sigma^2-80\sigma-193)}{45}\mu^2n^2 +$$

$$+ \frac{r^2}{\mu^2}\left[\frac{14\sigma^2+27\sigma+24}{3}(\mu^2+n^2)^2 + \frac{2(2\sigma^2+6\sigma+15)}{3}n^4 - \frac{11\sigma^3+53\sigma^2+81\sigma-5}{6}\mu^2n^2 - \right.$$

$$\left. - \frac{8}{3}\mu^4(\mu^2+n^2)\right] + \frac{r^4}{\mu^4}\left[\frac{2\sigma^2+3\sigma-4}{2}(\mu^2+n^2)^2 - 2(2\sigma^2+5\sigma+4)n^4 + \right.$$

$$\left. + \frac{\sigma^3-57\sigma^2-213\sigma-171}{8}\mu^2n^2\right].$$

Thus, the problem of the influence of normal pressure on the frequencies of natural oscillations is reduced to a determination of those values of $p$ and $\omega^2$ for which the condition of existence of non-trivial solutions is satisfied.

For this, we will examine a small dynamic disturbance of a shell due to force factors distributed over its surface. We will assume that the supporting shell has undergone the effect of these external forces. Now, the movement of the shell has the nature of natural oscillations around the initial stable equilibrium position. Subsequently, in determining the critical load for a nearly cylindrical shell, we will consider that its losses in stability signify that the frequencies of natural oscillations are equal to zero. Therefore, assuming that $\omega = 0$ in (2.2), where the expressions for $F_{1i}$, $F_{2i}$, and $p_1$ ($i = 0,1,2$) are determined by (2.3), (2.4) and (2.5) respectively, and considering the condition where $\mu^2 \ll n^2$, we can obtain the critical values for the pressure $\overline{p}_i = p_iR/2Eh$ for nearly cylindrical shells.

Since the component parameters of the pressure do not depend on $\lambda$, according to (1.4), we will make the factors for the corresponding powers of $\lambda$ in (2.2) equal to zero in order to determine them.

Having made each derivative by $n^2$ from the right-hand part of the equation equal to zero separately in the expressions obtained for $\overline{p}_i$($i = 0,1,2$), we can obtain the number of waves at which the shell in question loses stability.

For these values of $n$, /76

$$n_0 = 2{,}33\sqrt[8]{1-\sigma^2}\,\sqrt{R/L}\,\sqrt[4]{R/h};\quad n_1 = 0{,}91\sqrt[6]{\mu}\varepsilon^{-1/12}n_0;$$

$$n_2 = \frac{\sqrt{0{,}63\mu^{-1}+0{,}64\mu^3}}{3{,}7\cdot\varepsilon^{1/6}}n_0,\ (L=2l-\text{ the length of the shell}).$$

The corresponding critical forces are expressed by the following formulas:

$$\bar{p}_0^{cr} = \frac{2{,}59}{(1-\sigma^2)^{3/4}}\frac{h}{L}\sqrt{h/R};\quad \bar{p}_1^{cr} = 2{,}21\mu^{1/3}\varepsilon^{-1/12}\bar{p}_0^{cr};$$

$$\bar{p}_2^{cr} = \frac{3{,}7}{\mu\sqrt{\varepsilon}}\left[\sqrt{0{,}63+0{,}64\mu^4} = 1{,}63\,(3{,}7147+1{,}5\mu^2)\,\mu^2\sqrt{\varepsilon}\right]\bar{p}_0^{cr},$$

The total critical pressure is determined by (1.4) and is equal to

$$\bar{p}^{cr} = \bar{p}_0^{cr}\{1+2{,}21\mu^{1/3}\varepsilon^{-1/12}\lambda + \frac{3{,}7\lambda^2}{\mu\sqrt{3}}[\sqrt{0{,}63+0{,}64\mu^4}+ \\ +1{,}63\,(8{,}7147+1{,}5\mu^2)\,\mu^2\sqrt{\varepsilon}]\}, \qquad (2.6)$$

where $\bar{p}_0^{cr}$ is the critical value of the pressure for the perfect cylindrical shell.

The calculated formula obtained in (2.6) shows that the critical value of the pressure depends on the initial deviations in the form characterized by the parameter $\lambda$.

Calculations for a shell with parameters of $R$ = 11.7 cm, $l$ = 17.9 cm, $h$ = 0.045 cm, $\lambda$ = 0.1; 0.25; 0.5 were conducted with the aid of the simple dependence in (2.6).

The values of the critical pressures calculated according to (2.6) are rather close to the experimental values of T.D. Karimbayev, as we can see from the Table. (Discrepancy of 1 - 2% for the shells of Series I, and 13% for shells of Series II.)

| Series of Shells | $\lambda$ | $p_{exp}$ | $\bar{p}^{cr}$ |
|---|---|---|---|
| I | 0,1 | 0,051 | 0,065 |
| II | 0,25 | 0,083 | 0,211 |
| III | 0,5 | 0,161 | 0,676 |